# COURSEPACK

**SCHEME**

### The scheme is an overview of work-integrated learning opportunities and gets students out into the real world. This will give what a course entails.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Course Title** | Discrete Mathematics | | | | **Course Type** | | | Theory | | |
| **Course Code** | C1UC224T | | | | **Class** | | |  | | |
| **Instruction delivery** | **Activity** | **Credits** | **Credit Hours** | | **Total Number of Classes per Semester** | | | | **Assessment in Weightage** | |
| **Lecture** | **3** | **3** | |
| **Tutorial** | **0** | **0** | | **Theory** | **Tutorial** | **Practical**  **Practical** | **Self-study** | **CIE** | **SEE** |
| **Practical** | **0** | **0** | |
| **Self-study** | **0** | **0** | |
| **Total** | **3** | **3** | | 45 | 0 | 0 | 0 | 50% | 50% |
| **Course Lead** |  | | | **Course Coordinator** | Dr. Ananya Manas  Ms. Neelam Kumari | | | | | |
| **Names Course Instructors** | **Theory** | | | | **Practical** | | | | | |
|  | | | |  | | | | | |

**COURSE OVERVIEW**

The students who are successful in discrete mathematics will be able to generalize from a single instance of a problem to an entire class of problems. As it covers probabilities, trees, graphs, logic, mathematical thinking, and much more. This course is important in the sciences, for example understanding of DNA sequences in molecular biology, and useful in studying and describing objects and problems in all branches of computer science, such as computer algorithms, programming languages, cryptography (using in Data Security Council of India and IBM Security Guardium), and software development.

**PREREQUISITE COURSE**

|  |  |  |
| --- | --- | --- |
| **PREREQUISITE COURSE REQUIRED** | **NO** | |
| **If, yes please fill in the Details** | **Prerequisite course code** | **Prerequisite course name** |
| **NIL** | **NIL** |

**COURSE OBJECTIVE**

The objective of this course is to familiarize the prospective computer scientists with the techniques of

mathematical reasoning, logical thinking, abstract mathematical discrete structures so that they may apply

a particular set of mathematical facts in relevant situations.

## COURSE OUTCOMES (COs)

After the completion of the course, the student will be able to:

|  |  |
| --- | --- |
| **CO No**. | **Course Outcomes** |
| CO1 | Remember the concepts of sets, relations, functions, algebraic structures, the rule of inference for connecting and validating logical statements and basics concepts of graph theory. |
| CO2 | Explain the counting techniques and to solve various counting problems, concept of Posets and Lattices. |
| CO3 | Apply counting principles to determine probabilities and apply basic graph theory, minimal weighted spanning tree algorithms, graph coloring algorithms. |
| CO4 | Demonstrate and use the concept of graphs, trees, and related discrete mathematics, apply the methods from these subjects in problem solving. |

## BLOOM’S LEVEL OF THE COURSE OUTCOMES

Bloom's taxonomy is a set of hierarchical models used for the classification of educational learning objectives into levels of complexity and specificity. The learning domains are cognitive, affective, and psychomotor.

**THEORY**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| CO No. | Remember  **BTL1** | Understand **BTL2** | Apply  **BTL3** | Analyse **BTL4** | Evaluate **BTL2** | Create **BTL6** |
| 1 | 🗸 |  |  |  |  |  |
| 2 |  | 🗸 |  |  |  |  |
| 3 |  |  | 🗸 |  |  |  |
| 4 |  |  | 🗸 |  |  |  |

PROGRAM OUTCOMES (POs): AS DEFINED BY CONCERNED THE APEX BODIES

|  |  |
| --- | --- |
| **PO1** | **Engineering knowledge:** Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems. |
| **PO2** | **Problem analysis:** Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences. |
| **PO3** | **Design/development of solutions:** Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations. |
| **PO4** | **Conduct investigations of complex problems:** Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions. |
| **PO5** | **Modern tool usage:** Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations. |
| **PO6** | **The engineer and society:** Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues, and the consequent responsibilities relevant to the professional engineering practice. |
| **PO7** | **Environment and sustainability:** Understand the impact of the professional engineering solutions in societal and environmental contexts, demonstrate the knowledge of, and need for sustainable development. |
| **PO8** | **Ethics:** Apply ethical principles and commit to professional ethics, responsibilities, and norms of the engineering practice. |
| **PO9** | **Individual and Teamwork:** Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings. |
| **PO10** | **Communication:** Communicate effectively on complex engineering activities with the engineering community and with society, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions. |
| **PO11** | **Project management and finance:** Demonstrate knowledge and understanding of the engineering and management principles and apply these to one’s own work, as a member and leader in a team, to manage projects and in multidisciplinary environments. |
| **PO12** | **Life-long learning:** Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change. |

**PSO1:** Ability to work with emerging technologies in computing requisite to Industry 4.0

**PSO2:** Demonstrate Engineering Practice learned through industry internship to solve live

problems in various domains.

## COURSE ARTICULATION MATRIX

The Course articulation matrix indicates the correlation between Course Outcomes and Program Outcomes and their expected strength of mapping in three levels (low, medium, and high).

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **COs#/ POs** | **PO1** | **PO2** | **PO3** | **PO4** | **PO5** | **PO6** | **PO7** | **PO8** | **PO9** | **PO10** | **PO11** | **PO12** | **PSO1** | **PSO2** |
| CO1 | 3 | 3 | 1 | 1 |  |  |  |  |  |  |  |  | 2 | 2 |
| CO2 | 3 | 3 | 1 | 1 |  |  |  |  |  |  |  |  | 2 | 1 |
| CO3 | 3 | 2 | 1 | 1 |  |  |  |  |  |  |  |  | 1 | 1 |
| CO4 | 3 | 2 | 1 | 1 |  |  |  |  |  |  |  |  | 2 | 2 |

**Note:** 1-Low, 2-Medium, 3-High

## COURSE ASSESSMENT

The course assessment patterns are the assessment tools used both in formative and summative examinations.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Type of Course** | **CIE Weightage** | | | **End Term Exam (ETE) Weightage** |
| **IA-1** | **IA-2** | **Mid Term Exam** |
| Integrated (B) | 25 | 25 | 50 | 100 |
| Final Weightage | 25 | | 25 | 50 |
| **Total** | **100** | | | |

* Assignment, Quiz, Class test, SWAYAM/NPTEL/MOOCs and etc.

## COURSE CONTENT

|  |
| --- |
| **Content** |
| Syntax, Semantics, Validity and Satisfiability, Basic connectives and Truth Tables, Logical Equivalence, the laws of logic, Logical implication, Rules of inference, Normal form (CNF, DNF), Predicate logic, Universal and Existential quantifiers.  Proof Techniques: Some terminologies, Proof methods and strategies, Forward proof, Proof by contradiction, Proof by contraposition, Proof of necessity and sufficiency.  Counting Techniques: Basic counting techniques, inclusion, and exclusion, pigeon-hole principle, permutation, and combination  Operations and laws of sets, Cartesian product, binary relation, partial order relation, Equivalence relation, Functions, Bijective function, inverse and composition of function, size of a set, countable and uncountable set, Cantor’s diagonal argument and the power set theorem, Schroeder-Bernstein theorem.  Principles of Mathematical Induction: The well -Ordering principle, Recursive definition, prime numbers, greatest common divisor, Euclidean algorithm, the fundamental theorem of arithmetic.  Algebraic structures with one binary operation: Semi Group, Monoid, Groups, Subgroups, Congruence relation and quotient structures, Free and Cyclic Monoid and Groups, Cosets, Lagrange's theorem, Normal Subgroups, Permutation and Symmetric groups, Group Homomorphism, Algebraic structures with two binary operations: Ring, Integral domain, and Field.  Graphs and their properties, degree, connectivity, path cycle, sub graphs, isomorphism, Eulerian and Hamiltonian walks, Graph coloring, and planer graphs, Trees: Definitions, properties, and examples.  Partial order sets, Hasse diagram, Lattices, and its properties of lattices |

**LESSON PLAN FOR THEORY COURSES (15 weeks \* 3 Hours = 45 Classes)**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Lec. | Topic | L/T/P | Skills | Competency |
| 1. 1 | Propositions, logical operators, Truthtables (TT) | L | 1.To frame the logical statements  2. To frame the arguments with the help of mathematical and logical expressions.  3. To check the validity of arguments  4. Determine the logical equivalency of the compound statements. | Logical thinking |
| 1. 2 | Conditional statements, converse, contrapositive, inverse, biconditionals | L |
| 1. 3 | TT of compound propositions, precedence of logical operators | L |
| 1. 4 | Tautology, contradiction, logical equivalence, laws, Satisfiability | L |
| 1. 5 | Predicates, quantifiers | L |
| 1. 6 | Rules of Inference | L |
| 1. 7 | Normal form- CNF, DNF | L |
| 1. 8 | Proof Techniques: Some terminologies, Forward proof, Proof by contradiction, Proof by contraposition, Proof of necessity and sufficiency. | L |
| 1. 9 | Basic counting techniques, Inclusion and exclusion rule | L |
| 1. 10 | Pigeon-hole principle, Generalized pigeon-hole principle | L |
| 1. 11 | Permutation | L |
| 1. 12 | Combination | L |
| 1. 13 | Set, Venn diagram, Subsets, size of the set, power set, Cartesian Product | L | 1.To understand sets and perform operations and algebra on sets determine the properties of relation and functions, identify equivalence and partial ordered relation. Determining the domain and range | Demonstrate the understanding of sets, relations and functions and use them for counting techniques |
| 1. 14 | Set operations, set identities, Generalized Unions and Intersections, Computer Representation of Sets | L |
| 1. 15 | Relations, properties of relations, Combining Relations | L |
| 1. 18 | Equivalence Relation and Classes, partition | L |
| 1. 19 | Functions, one-one, onto, bijection | L |
| 1. 20 | Inverse and composition functions, floor & ceiling function | L |
| 1. 21 | Cardinality of the Set, Countable and Uncountable set, Cantor’s diagonal theorem the power set theorem, Schroeder-Bernstein theorem | L |
| 1. 22 | Mathematical Induction, Strong induction, the well ordering Principle | L |
| 1. 23 | Recursive Relation | L |
| 1. 24 | Prime Number and Greatest common divisor of two numbers | L |
| 1. 25 | Euclidean algorithm | L |
| 1. 26 | The fundamental theorem of arithmetic | L |
| 1. 27 | Introduction of Algebraic Structures: Monoids and semigroups. | L | Analyze the basic structures of algebraic structures | Study of symmetry |
| 1. 28 | Introduction of Algebraic Structures: Groups | L |
| 1. 29 | Groups, Subgroups, | L |
| 1. 30 | Congruence relation | L |
| 1. 31 | Coset, Lagrange's theorem. | L |
| 1. 32 | Coset, Lagrange's theorem. | L |
| 1. 33 | Normal Subgroups, Permutation & Symmetric groups, Group Homomorphism | L |
| 1. 34 | Normal Subgroups, Permutation & Symmetric groups, Group Homomorphism | L |
| 1. 35 | Algebraic structures with two binary operations: Ring, Integral domain, Field. | L |
| 1. 36 | Algebraic structures with two binary operations: Integral domain, Field. | L |
| 1. 37 | Graph & graph models | L | 1.Basic terminology of graph, tree, and path  2.Analyze the relationships between different objects or entities  3.To represent the interactions between different objects | Able to explain basic terminology of a graph Identify Euler and Hamiltonian cycle Represent graphs using adjacency matrices |
| 1. 38 | Graph Terminology and Special Types of Graphs | L |
| 1. 39 | Representing Graphs and Graph Isomorphism | L |
| 1. 40 | Connectivity, Euler and Hamilton Paths | L |
| 1. 41 | Planar Graphs, Graph Coloring | L |
| 1. 42 | Introduction to Trees& applications | L |
| 1. 43 | Tree Traversal, Spanning Trees, Minimum Spanning Trees | L |
| 1. 44 | Partial ordered set | L |
| 1. 45 | Combination of partial order sets, Hasse diagram | L |
| 1. 46 | Lattices: Definition, Properties of lattices – Bounded, Complemented | L |
| 1. 7 | Modular and Complete lattice | L |

**BIBLIOGRAPHY**

### **Text Book**

### T1: Kenneth H. Rosen, *Discrete Mathematics and Its Applications, 8th Edition*, McGraw Hill Education, 2021.

* + **Reference Books**

R1: J P Trembley, R Manohar*, Discrete Mathematical Structures with Applications to Computer Science, 1st Edition***,** McGraw-Hill Education, 2017.

R2: Semyour Lipschutz,Marc Lipson, Varsha H. Patil*, Discrete Mathematics (Schaum's Outlines, 3rd Edition,* McGraw Hill Education, 2017.

Journals/Magazines/Govt. Reports/Gazatte/Industry Trends

1. [**https://www.sciencedirect.com/journal/discrete-mathematics**](https://www.sciencedirect.com/journal/discrete-mathematics)
2. [**https://www.degruyter.com/journal/key/dma/html?lang=en**](https://www.degruyter.com/journal/key/dma/html?lang=en)

**Webliography :** [www.mhhe.com/rosen](http://www.mhhe.com/rosen)

[**https://nptel.ac.in/courses/106106183**](https://nptel.ac.in/courses/106106183)

* + **SWAYAM/NPTEL/MOOCs Certification:**

[**https://nptel.ac.in/courses/106106183**](https://nptel.ac.in/courses/106106183)

## PROBLEM-BASED LEARNING

Exercises in Problem-based Learning (Assignments)

|  |  |  |
| --- | --- | --- |
| **S.No** | **Questions** | **Blooo’s level** |
|  | W Which of these sentences are **propositions**? What are the **truth values** of those that are propositions? **a)** 2 + 3 = 5. **b )** Answer this question. | K1 |
|  | Check whether these statements are wff or not:(a) (p˅q) ∧∼r (b) p˅q∧r | K2 |
|  | Determine whether these **biconditionals** are true or false.  a) 2 + 2 = 4 if and only if 1 + 1 = 2. b) 1 + 1 = 2 if and only if 2 + 3 = 4. | K2 |
|  | Let *P(x)* be the statement “*x* = *x*2.” If the domain consists of the integers, what are these truth values?  **a)** *P(*1*)* **b)** ∃*xP(x)* (11/53/KR) | K2 |
|  | Determine the truth value of each of these statements if the domain consists of all integers.  **a)** ∀*n(n* + 1 *> n)* **b)** ∃*n(*2*n* = 3*n)* | K2 |
|  | Use **De Morgan’s laws** to find the negation of each of the following statements. **a)** Jasbir is rich and happy **b)** Rain will bicycle or run tomorrow. | K2 |
|  | Show that *(p* → *q)* ∧ *(q* → *r)* → *(p* → *r)* is a **tautology** | K2 |
|  | Give a direct proof of the theorem “If n is an odd integer, then n2 is odd.” | K2 |
|  | Give a proof by contradiction of the theorem “If 3n+2 is odd, then n is odd”. | K2 |
|  | For each of these relations on the set {1,2,3,4}, decide whether it is reflexive, whether it is symmetric, whether it is anti-symmetric, and whether it is transitive.   1. {(2,2), (2,3), (2,4),(3,2), (3,3), (3,4)} 2. {(1,1),(1,2),(2,1),(2,2),(3,3),(4,4)} 3. {(2,4),(4,2)} 4. {(1,2),(2,3),(3,4)}   {(1,1),(2,2),(3,3),(4,4)} | K2 |
|  | Let R={(1,2),(1,3),(2,3),(2,4),(3,1)} and S ={(2,1),(3,1),(3,2),( 4,2)} be relations defined on{1,2,3,4}. Find SoR and RoS. | K2 |
|  | Define Sub-Group of a Group. | K1 |
|  | Define order of Group and Order of an element of Group. | K2 |
|  | Define i) a permutation, ii) a symmetric group. | K2 |
|  | Give an example of a semi-group which is not a group. | K2 |
|  | Let Find and express it as a product of disjoint cycles. State whether | K2 |
|  | Show that every subgroup of an Abelian/Cyclic group is Normal. | K2 |
|  | Consider the ring of integers modulo 10.  (a) Find the units of .  (b) Find −3, −8, and . | K3 |
|  | Prove that *F* ={ *a* + 2 | *a, b* integers} is an integral domain but not a field. | K3 |
|  | Show that is an integral Domain under addition and multiplication modulo 10. | K3 |
|  | How many vertices and how many edges do  graphs have? | K1 |
|  | How many sub graphs with at least one vertex does have? | K2 |
|  | For which values of and  is  is regular? | K1 |
|  | Is every zero-one square matrix that is symmetric and has zeros on the diagonal the adjacency matrix of a simple graph? | K1 |
|  | What is the sum of the entries in a column of the adjacency matrix for an undirected graph? For a directed graph? | K2 |
|  | Show that a connected multigraph with at least two vertices has an Euler circuit iff each of its vertices has an even degree. | K3 |
|  | Show that  has a Hamilton circuit whenever | K3 |
|  | What is the chromatic number of ? | K2 |
|  | Prove that the number of vertices in a full binary tree is always odd. | K1 |
|  | Suppose that a connected planar simple graph has 20 vertices, each of degree 3. In to how many regions does a representation of this planar graph split the plane? | K3 |
|  | State and prove Euler’s formula. | K3 |
|  | Show that the “greater than or equal” relation (≥) is a partial ordering on the set of integers. | K1 |
|  | In the poset, are the integers 3 and 9 comparable? Are 5 and 7 comparable? | K1 |
|  | Determine whether the relations represented by these zero-one matrices are partial orders.   1. b) | K1 |
|  | Find the dual of Poset | K1 |
|  | Draw the Hasse diagram representing the partial ordering on | K3 |
|  | Let S= {1, 2, 3, 4}. With respect to the lexicographic order based on the usual “less than” relation, find all pairs in less than . | K2 |
|  | Draw the Hasse diagram for divisibility on the set. | K3 |
|  | Define Lattice. | K1 |
|  | Which elements of the poset are maximal, and which are minimal? | K2 |
|  | Let S be a set. Determine whether there is a greatest element and a least element in the poset . | K2 |
|  | Find the greatest lower bound and the least upper bound of the set and , if they exist, in the poset. | K2 |
|  | Is the poset a lattice? | K2 |
|  | Determine whether (P(S), ) is a lattice where S is a set. | K2 |
|  | Show that every totally ordered set is a lattice. | K3 |

## STUDENT-CENTERED LEARNING (SELF-LEARNING TOWARDS LIFE-LONG-LEARNING)

Self-Learning (it’s a typical course-based project to be carried out by a whole class in groups of four students each; they should exhibit higher level BTLs)

The students, in a group, are expected to conceive an idea based on the content (objectives/outcomes) and apply the suitable knowledge to demonstrate their learning.

A list of 30-40 project statements can be offered to the students to choose or students can conceive their own ideas (teamwork), design and develop the product/process/service and implement the same.

## COURSE-BASED PROJECT (Psychomotor skills)

To enhance their skill set in the integrated course, the students are advised to execute course-based

**design projects**. Some sample projects are given below:

|  |  |  |
| --- | --- | --- |
| **S. No.** | **Suggested Projects** | BTL |
| 1 | **Game Theory:** Designing interesting games and/or finding winning strategies for known games. Describe the game in terms of graphs, what are you trying to achieve or avoid?   1. What games that you know can be studied in terms of graphs? Is Tic-Tac-Toe an example of this? What is the graph? What are you trying to avoid? You can also study Tic-Tac-Toe generalizations. 2. Find out how the Game of Dim is played and study it from the graph theory point of view. | L6 |
| 2 | **Number Theory:** Understand divisibility criteria. Develop divisibility criteria for "nontraditional" primes, like 7 or 13. Can you explain why these are not usually mentioned in the regular literature? | L5 |
| 3 | **Scheduling Problems:** When organizing a conference or event, how do you schedule the talks according to participant and room restrictions? How are final exams, spelling bee competitions, sports competitions scheduled at your school? How do you make sure that all or must participants will be able to attend their events or exams? | L6 |
| 4 | **Power in games:** Are there student elections at your school? Do you vote on certain school decisions? How much power do you have in an election where each department has one vote but some departments are larger than others? What kind of majority is needed? | L6 |
| 5 | **Matchings, SDRs, and Stable Marriage Problems:** How to form teams in your class according to classmates’ preferences (lists of preferences are provided by each student)? Look for other variations on this situation. | L6 |
| 6 | **Algebraic representations of graphs:** Study the adjacency matrix of a graph. How can you find the number of edges, the degrees, the number of triangles, etc., without drawing the graph. Just by using the entries of the matrix. | L5 |
| 7 | **Latin squares:** Study Latin squares and their relationship to Sudoku. How many Latin squares are there of a fixed size under certain given restrictions? Come up with easier but still interesting Sudoku variations. | L6 |

1. **SELF-LEARNING THROUGH MOOCs (Cognitive Skills):** Certification

https://www.coursera.org/learn/discrete-mathematics